# The Relationship Between Detection Algorithms for Hyperspectral and Radar Applications

Nirmal Keshava, Stephen M. Kogon, Dimitris Manolakis

March 14, 2001

ASAP Conference
MIT Lincoln Laboratory
Lexington, MA  02420

| # Report Documentation Page | *Form Approved*<br>*OMB No. 0704-0188* |
|---|---|



| 1. REPORT DATE | 2. REPORT TYPE | 3. DATES COVERED |
|---|---|---|
| **14 MAR 2001** | **N/A** | **-** |

| 4. TITLE AND SUBTITLE | |
|---|---|
| **The Relationship Between Detection Algorithms for Hyperspectral and Radar Applications** | 5a. CONTRACT NUMBER |
| | 5b. GRANT NUMBER |
| | 5c. PROGRAM ELEMENT NUMBER |
| 6. AUTHOR(S) | 5d. PROJECT NUMBER |
| **Nirmal Keshava; Stephen M. Kogon; Dimitris Manolakis** | 5e. TASK NUMBER |
| | 5f. WORK UNIT NUMBER |
| 7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)<br>**MIT Lincoln Laboratory 244 Wood Street Lexington, MA 02420-9185** | 8. PERFORMING ORGANIZATION REPORT NUMBER |
| 9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) | 10. SPONSOR/MONITOR'S ACRONYM(S) |
| | 11. SPONSOR/MONITOR'S REPORT NUMBER(S) |

12. DISTRIBUTION/AVAILABILITY STATEMENT

**Approved for public release, distribution unlimited**

13. SUPPLEMENTARY NOTES

**See ADM001263 for entire Adaptive Sensor Array Processing Workshop., The original document contains color images.**

14. ABSTRACT

**See Briefing Charts.**

15. SUBJECT TERMS

| 16. SECURITY CLASSIFICATION OF: | | | 17. LIMITATION OF ABSTRACT | 18. NUMBER OF PAGES | 19a. NAME OF RESPONSIBLE PERSON |
|---|---|---|---|---|---|
| a. REPORT | b. ABSTRACT | c. THIS PAGE | | | |
| **unclassified** | **unclassified** | **unclassified** | **UU** | **22** | |

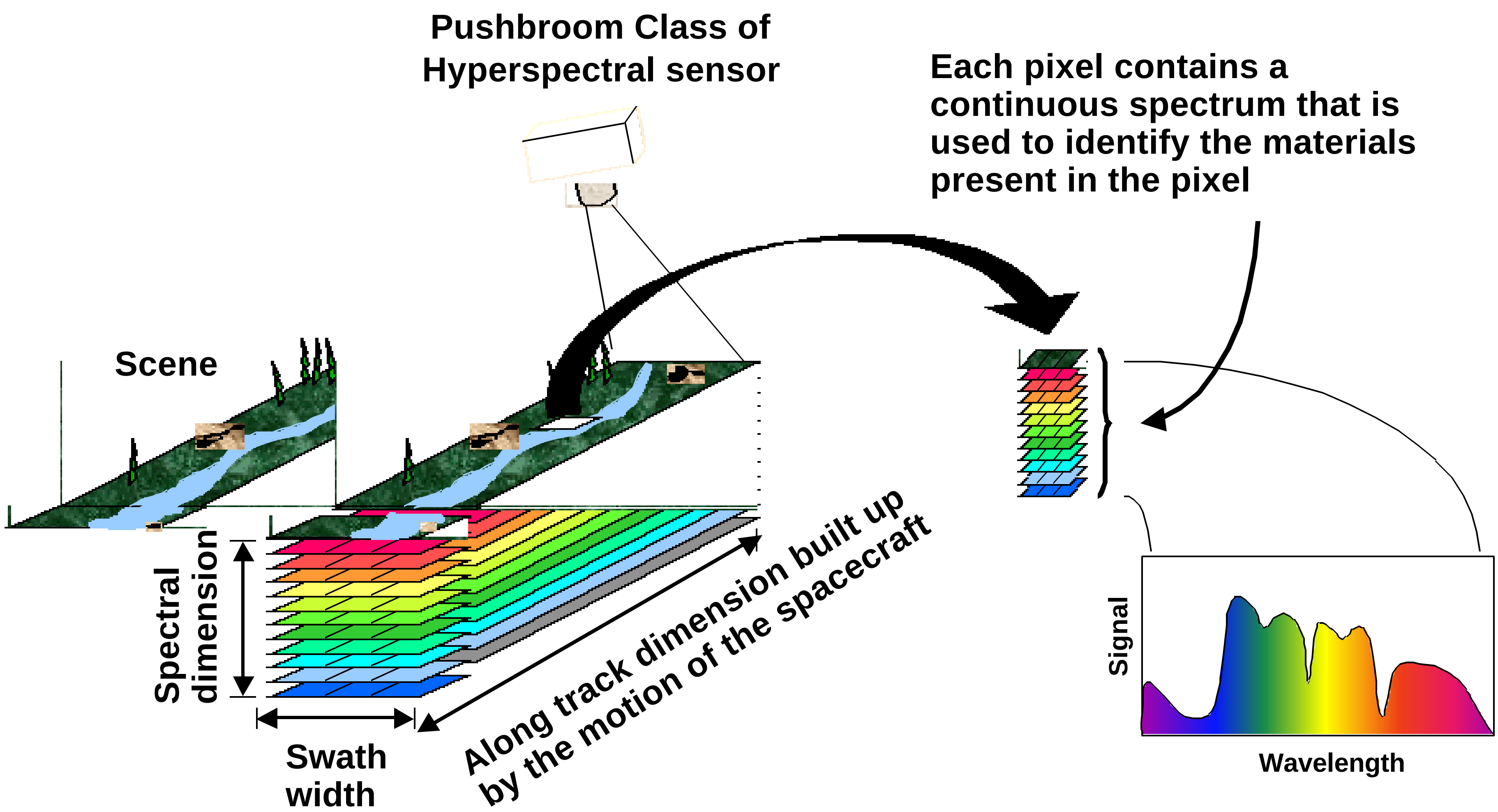
# Objective

- Overview of hyperspectral sensing
- Demonstrate how and why detection algorithms for hyperspectral imagery are related to detection algorithms for MTI radar
  - Similar physical assumptions
  - Common signal model
- Illustrate detection in hyperspectral imagery with real data and familiar detectors

# Outline

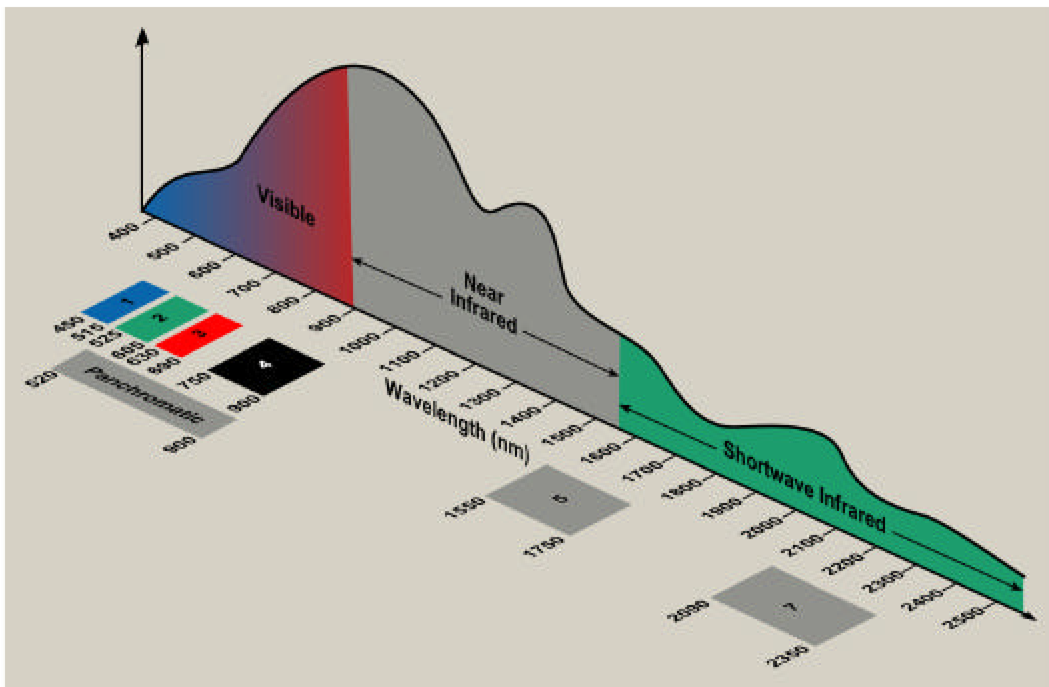
- Introduction to hyperspectral sensing
- Signal models
- Detection models
- Hyperspectral detection results
- Conclusion

# Hyperspectral Imaging (HSI) Concept



# Hyperspectral Sensing

- **Hyperspectral imaging (HSI) is a form of *passive* imaging**
  - **Extension of multispectral sensing (e.g., Landsat)**
  - **Hundreds of contiguous, real-valued spectral bands**
  - **Spatial resolution is a function of Instantaneous Field of View (IFOV) and altitude**

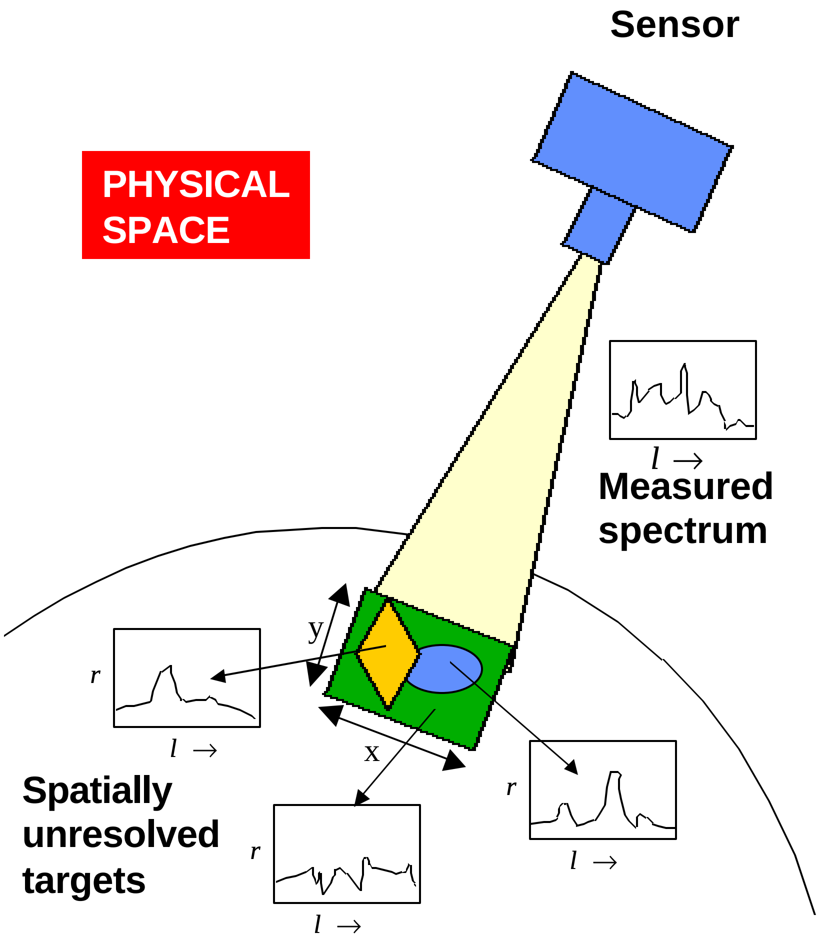


# Outline

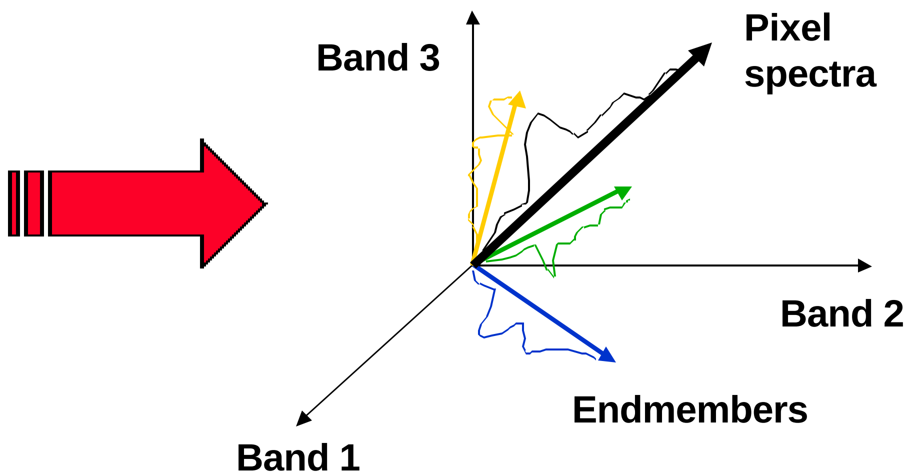
- Introduction to hyperspectral sensing
- Signal models
  - Hyperspectral sensing
  - MTI radar
- Detection models
- Hyperspectral detection results
- Conclusion

# Modeling of Spatially Unresolved (Mixed) Pixels

**PHYSICAL SPACE**




**DATA SPACE**




**SIGNAL PROCESSING**

- Unmixing
  - Find endmembers
  - Compute abundances
- Classification
- Detection

# Linear Mixing Model (LMM)

## Target and Background Modeling

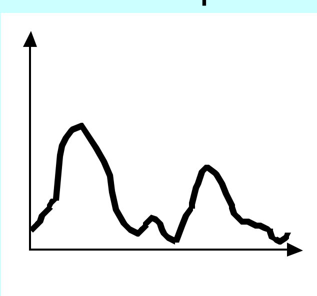
**LINEAR MIXING MODEL**

*abundance*

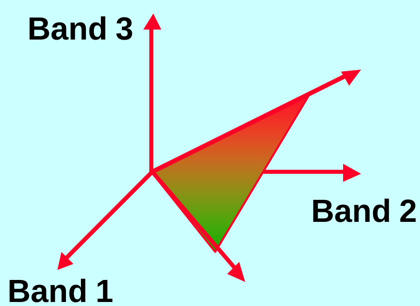
**Test pixel** $$\boldsymbol{x} = \sum_{k=1}^{P_T} a_k \boldsymbol{s}_k + \sum_{k=P_T+1}^{P_T+P_B} a_k \boldsymbol{s}_k + n$$

*end member*

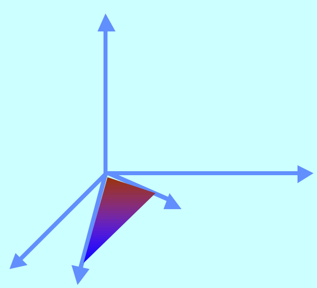
Measured spectrum



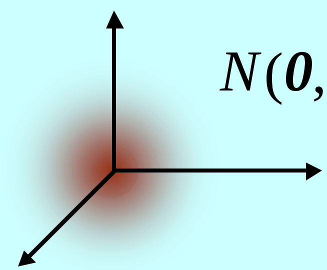
Target subspace




Background subspace

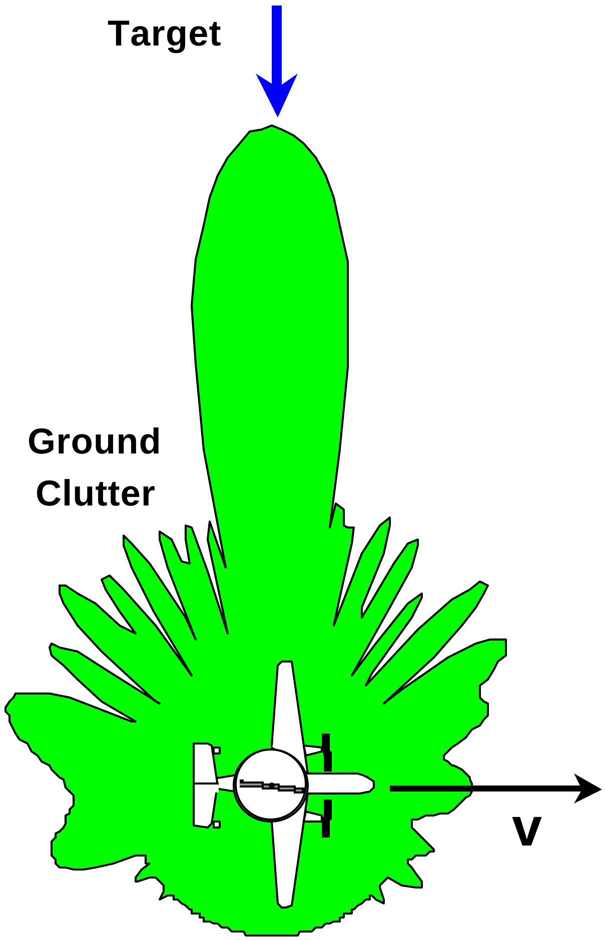


Noise hyper-sphere

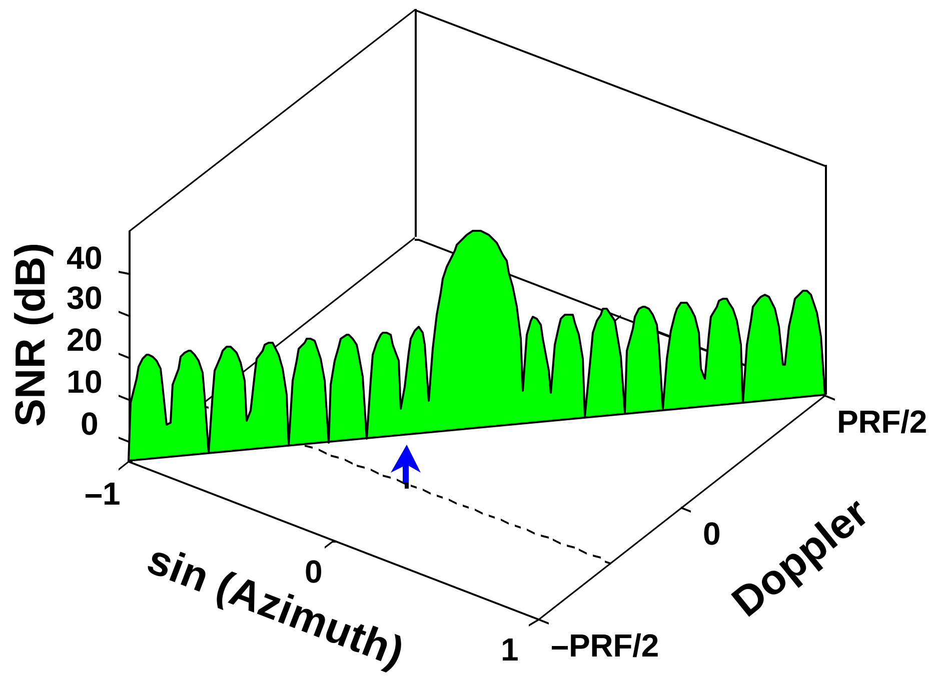


$N(\boldsymbol{0}, \sigma_w^2 \boldsymbol{I})$

# MTI Radar




Surveillance Radar

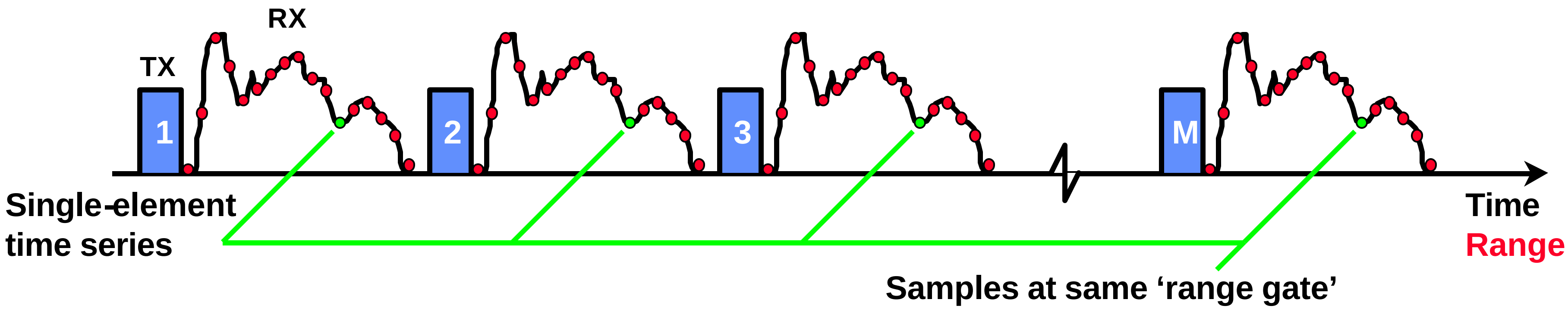


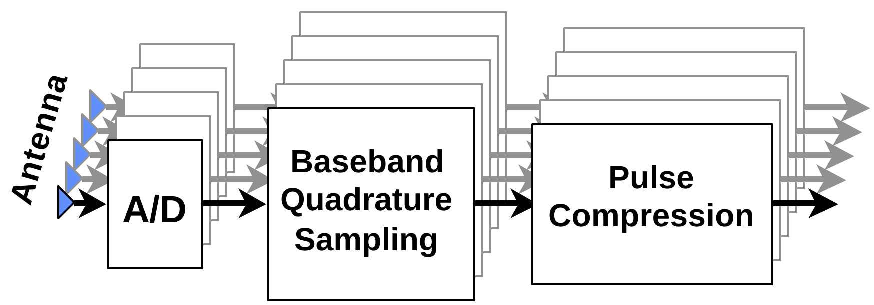

Two-dimensional filtering required to cancel interference

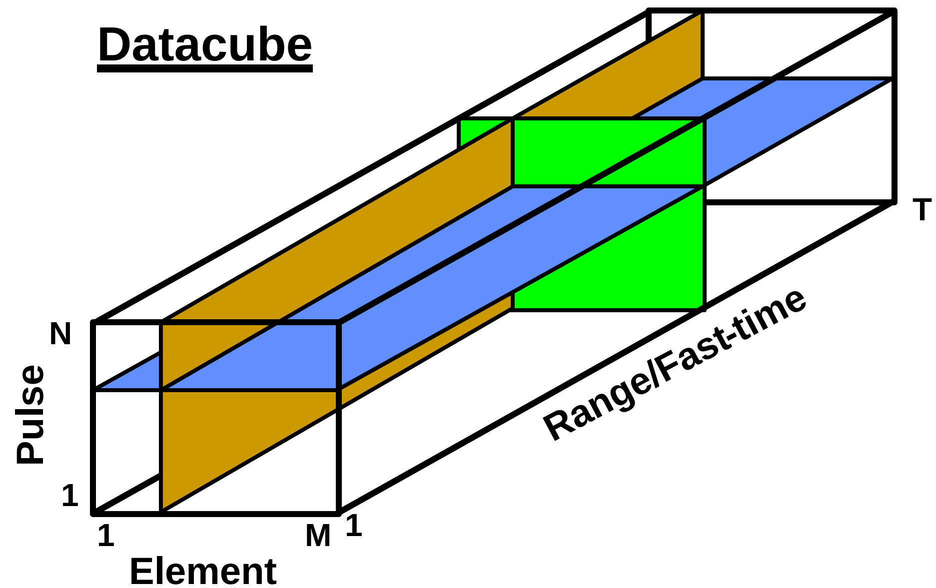
Space-Time Adaptive Processing

**(STAP)**

# Pulsed Radar Datacube




| Measurement | | Physical Quantity |
|---|---|---|
| Pulse | → | Doppler (velocity) |
| Element | → | Angle |
| Fast-time | → | Range |

# STAP Radar Signal Model

- **Space-time snapshot for single target**

$$\mathbf{x} = \mathbf{t} + \mathbf{c} + \mathbf{n} \qquad\qquad \mathbf{t} = a\,\mathbf{v}(f\,,\,f\,)$$

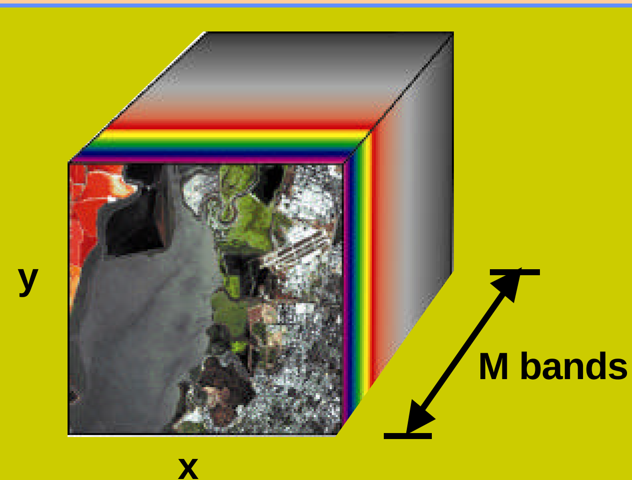
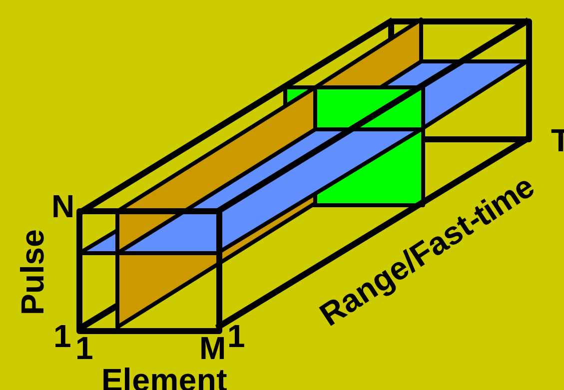
- $\mathbf{v}(f\,,\,f\,)$ **is called the space-time <u>steering vector</u>**

- **Space-time interference (clutter, noise) covariance is**

$$\mathbf{R} = E\left\{(\mathbf{c}+\mathbf{n})\,(\mathbf{c}+\mathbf{n})^H\right\} = \mathbf{R}_c + \mathbf{R}_n$$

# Hyperspectral Imaging and MTI Radar

## Summary of Properties

| | Hyperspectral Imaging | MTI Radar |
|---|---|---|
| System | • Passive, incoherent sensing<br>• Resolution is a function of detector IFOV and altitude | • Active, coherent sensing<br>• Resolution is a function of signal bandwidth and aperture length |
| Signal Model | • LMM assumes distinct spectra mix linearly<br>• Real spectra are sum of <u>endmembers</u> weighted by <u>abundances</u><br>$\mathbf{x} = a\mathbf{s} + \mathbf{b} + \mathbf{n}$ | • Components add linearly to yield received signal<br>• Complex array measurements are sum of <u>steering vectors</u> weighted by <u>RCS</u> values<br>$\mathbf{x} = a\,\mathbf{v} + \mathbf{c} + \mathbf{n}$ |
| Data Cube | <br> | <br> |

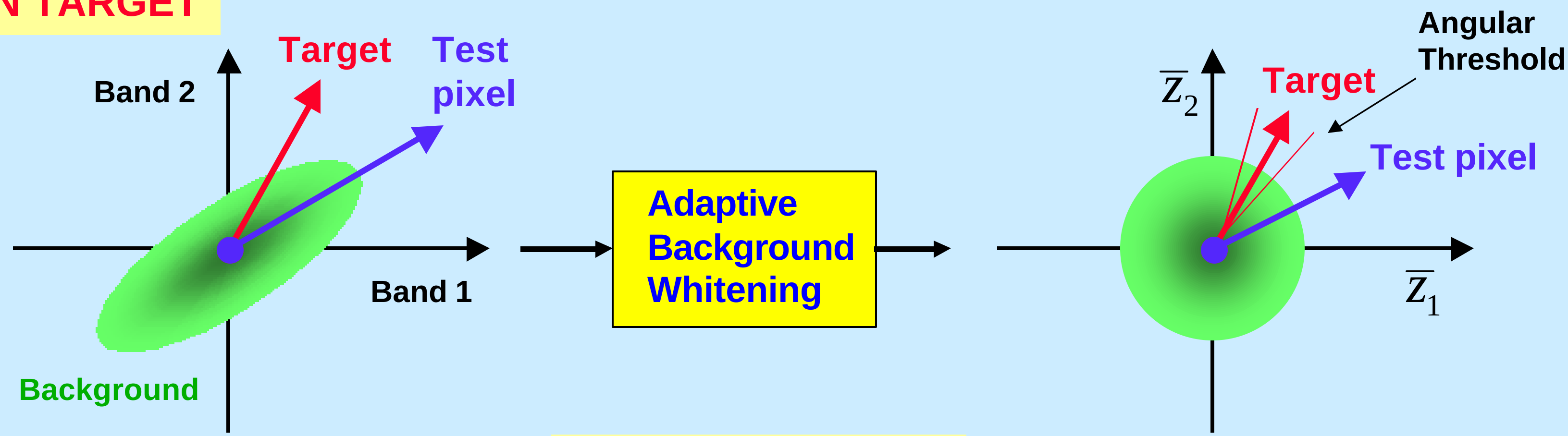
# Outline

- Introduction to hyperspectral sensing
- Signal models
- Detection models
  - Hyperspectral sensing
  - MTI radar
- Hyperspectral detection results
- Conclusion

# Adaptive HSI Detection

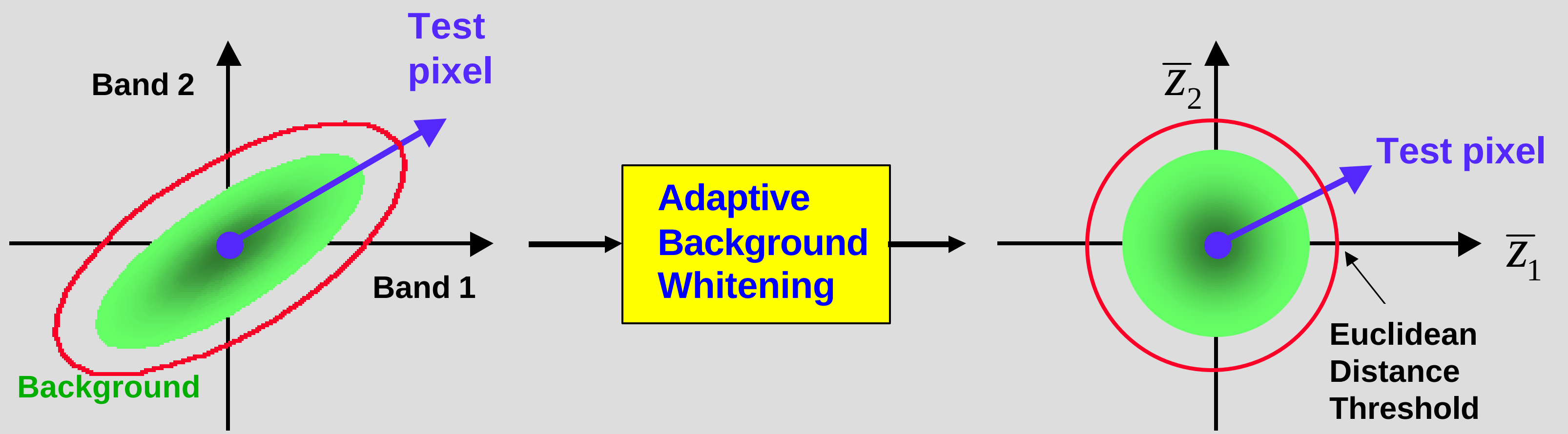
## Known and Unknown Targets

**KNOWN TARGET**




$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{x}(n)\boldsymbol{x}^T(n)$$

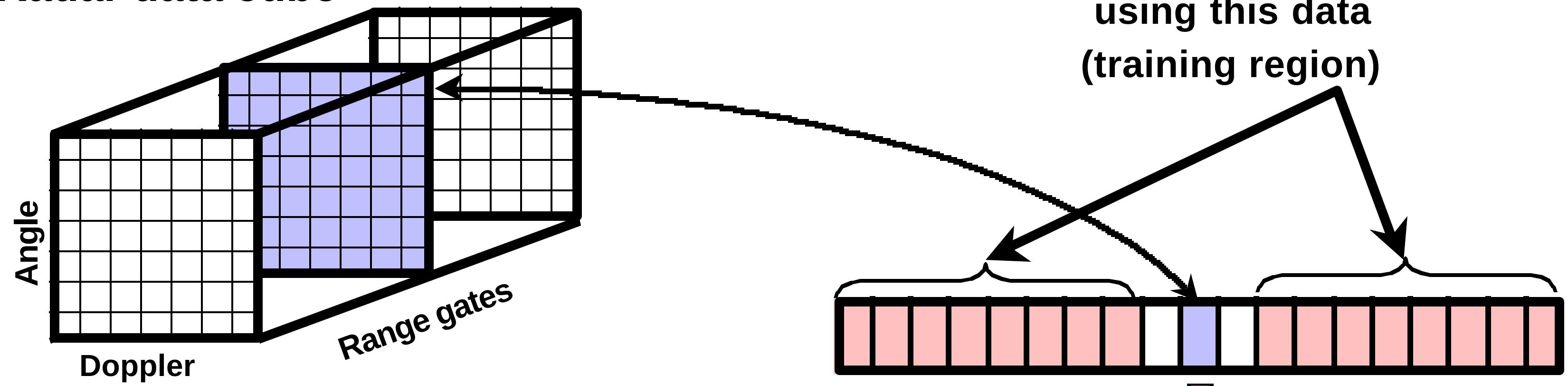
**UNKNOWN TARGET**



# Adaptive Detection in STAP Radar

**Radar data cube**

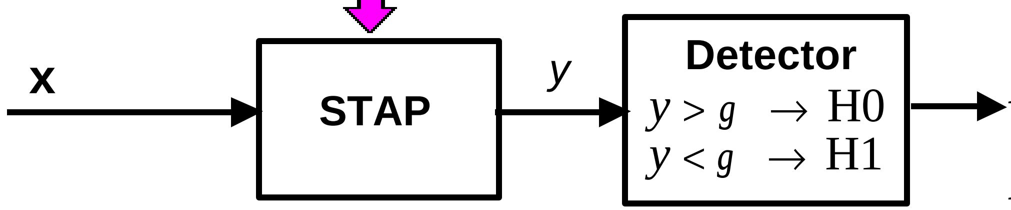
**Estimate interference using this data (training region)**




$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(k)\,\mathbf{x}(k)^H$$

**Estimate STAP Weights**

$$\mathbf{w} = \hat{\mathbf{R}}^{-1}\mathbf{v}(f\,,\,f\,)$$




**Hypothesis Testing:**

$H0: \quad \mathbf{x} = \mathbf{t} + \mathbf{c} + \mathbf{n}$ **Target present**

$H1: \quad \mathbf{x} = \mathbf{c} + \mathbf{n}$ **No target**

# Replacement and Additive Target Models

- Hyperspectral detection has <u>replacement</u> targets

$$H0: \quad \mathbf{x} = \mathbf{b} + \mathbf{n}$$

$$H1: \quad \mathbf{x} = f\,\mathbf{t} + (1 - f)\mathbf{b} + \mathbf{n}$$

- Interference statistics
  - Varies with $f, 0 \le f \le 1$
  - Target displaces background

- Detection results
  - Insufficient target data for ROC curves
  - No theoretical models

- MTI radar detection has <u>additive</u> targets

$$H0: \quad \mathbf{x} = \mathbf{c} + \mathbf{n}$$

$$H1: \quad \mathbf{x} = \mathbf{t} + \mathbf{c} + \mathbf{n}$$

- Interference statistics
  - Independent of target
  - Measure locally

- Detection results
  - ROC curves indicate $P_D/P_{FA}$ values
  - Theoretical models for target

# Comparison of HSI and MTI Detection

| | Hyperspectral Imaging | MTI Radar |
|---|---|---|
| Task | • Known target<br>– Detect target spectrum amid background<br>• Unknown target<br>– Detect pixels anomalous from background | • Moving target<br>– Detect Doppler effect at specific range and angle<br>– Use data after pulse compression |
| Covariance | • Interference covariance estimated from sample pixels<br>– Dimension equals number of bands (~ 100--200)<br>– Can use subset of bands | • Interference covariance estimated from local subset of pulse/element/range measurements<br>– Better estimate<br>– Avoids non-stationarity |
| Strategy | • Replacement target model<br>• Known target<br>– Measure spectral angle<br>• Unknown target<br>– Measure magnitude | • Additive target model<br>• Moving target<br>– Exploit coherency through beamforming and Doppler filtering<br>– RCS and velocity are key parameters for target visibility |

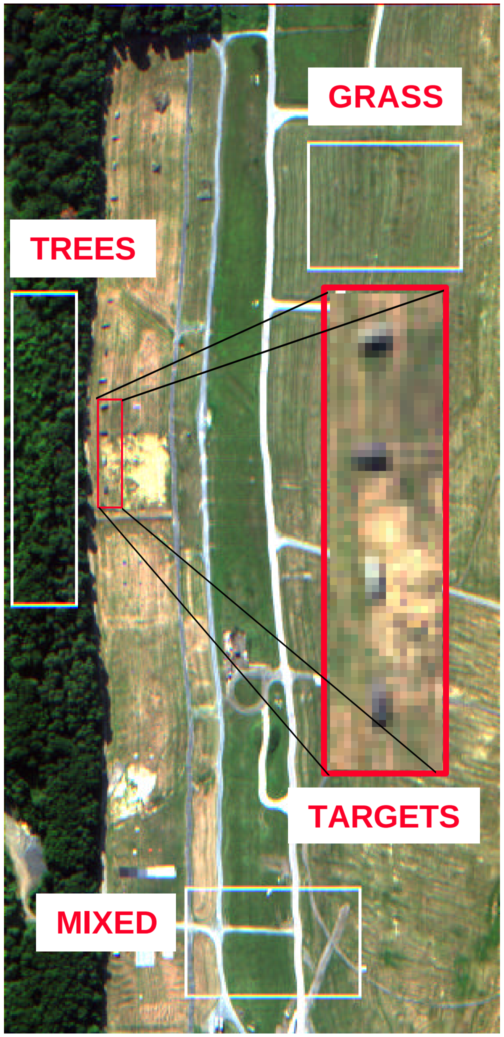
# Outline

- Introduction to hyperspectral sensing
- Signal models
- Detection models
- Hyperspectral detection results
  - Detection taxonomy
  - Sub-pixel target detection
- Conclusion

# Taxonomy of Hyperspectral Detectors

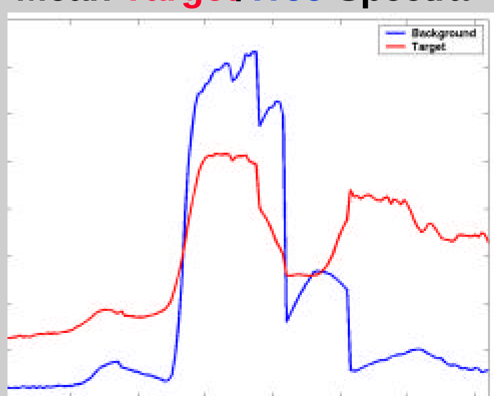
| Noise model | Signal model | Available data | Test statistic $T(x)$ | References | Comments |
|---|---|---|---|---|---|
| $\mathbf{R}$ = completely unknown interference (unstructured) | $\mathbf{s} = a\mathbf{s}_t$ known direction | $x$ = test measurement<br>$\{x_n\}_1^N$ = "signal-free" training data | $\dfrac{\lvert \mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{x}\rvert^2}{(\mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{s}_t)(1+\mathbf{x}^T\bar{\mathbf{R}}^{-1}\mathbf{x})}$ | Generalized Likelihood Ratio Test (GLRT)<br>Kelly (1986) | |
| | | $\bar{\mathbf{R}} = \sum_{n=1}^{N}\mathbf{x}_n\mathbf{x}_n^T$<br>$\hat{\mathbf{R}} = \dfrac{1}{N}\bar{\mathbf{R}}$ | $\dfrac{\lvert \mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{x}\rvert^2}{\mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{s}_t}$ | Adaptive Matched Filter (AMF)<br>Robey et al (1992)<br>Chen and Reed (1991) | $T_{CEM}(\mathbf{x}) = \dfrac{\mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{x}}{\mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{s}_t}$ |
| | | | $\dfrac{\lvert \mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{x}\rvert^2}{(\mathbf{s}_t^T\bar{\mathbf{R}}^{-1}\mathbf{s}_t)(\mathbf{x}^T\bar{\mathbf{R}}^{-1}\mathbf{x})}$ | Adaptive Coherence Estimator (ACE)<br>Conte et al (1995)<br>Scharf and McWhorter (1996) | $\mathbf{x} = \bar{\mathbf{R}}^{-1/2}\mathbf{x},\ \mathbf{s}_t = \bar{\mathbf{R}}^{-1/2}\mathbf{s}_t$<br>$\cos\theta = \dfrac{\lvert s_t^T x\rvert}{\lVert s_t\rVert\lVert x\rVert}$<br>$\mathbf{R} = \sigma^2\mathbf{I} \Rightarrow$ SAM |
| | $s = \sum_{k=1}^{P} a_k s_k = Sa$<br>$1 \le P \le M$ | | $\dfrac{\mathbf{x}^T\bar{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^T\bar{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^T\bar{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^T\bar{\mathbf{R}}^{-1}\mathbf{x}}$ | Kelly (1987,1989);<br>$P = M \Rightarrow$ unknown deterministic target, Reed-Yu (1990) | $P = 1 \Rightarrow$ GLRT<br>$P = M \Rightarrow$<br>$T(\mathbf{x}) = \mathbf{x}^T\bar{\mathbf{R}}^{-1}\mathbf{x}$<br>Simplicity $\Rightarrow \mathbf{S} \equiv \mathbf{I}_M$ |
| $\mathbf{R} = \sigma_w^2\mathbf{I} + \sum_{k=1}^{Q}\mathbf{z}_k\mathbf{z}_k^T$<br>structured interference | $\mathbf{s} = a\mathbf{s}_t$ | $x$ = test measurement<br>$\mathbf{S} \equiv [\mathbf{s}_1\ \mathbf{s}_2 \dots \mathbf{s}_P]$<br>$\mathbf{Z} \equiv [\mathbf{z}_1\ \mathbf{z}_2 \dots \mathbf{z}_Q]$ | $\mathbf{S} = \mathbf{s}_t \Rightarrow$<br>$\hat{a} = \dfrac{\mathbf{s}_t^T\mathbf{P}_Z^{\perp}\mathbf{x}}{\mathbf{s}_t^T\mathbf{P}_Z^{\perp}\mathbf{s}_t}$ | Classical F-test for linear statistical models;<br>OSP: Harsanyi-Chang (1994) | Orthogonal subspace projection (OSP):<br>$T(\mathbf{x}) = \mathbf{s}_t^T\mathbf{P}_Z^{\perp}\mathbf{x}$ |
| | $s = \sum_{k=1}^{P} a_k s_k = Sa$<br>$1 \le P \le M$ | | $T'(\mathbf{x}) = \dfrac{\mathbf{x}^T\mathbf{P}_Z^{\perp}\mathbf{P}_G\mathbf{P}_Z^{\perp}\mathbf{x}}{\mathbf{x}^T\mathbf{P}_Z^{\perp}\mathbf{P}_G^{\perp}\mathbf{P}_Z^{\perp}\mathbf{x}}$<br>$\mathbf{P}_G \equiv \mathbf{G}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T$<br>$\mathbf{G} \equiv \mathbf{P}_Z^{\perp}\mathbf{S}$ $\quad \mathbf{P}_G^{\perp} \equiv \mathbf{I} - \mathbf{P}_G$ | Classical F-test for linear statistical models; Signal processing interpretations<br>Matched Subspace Detector (MSD), Scharf-Friedlander (1994) | $T(x) = \dfrac{\frac{T'(x)}{P}}{M-P-Q}$ |

# Hyperspectral Detection Results

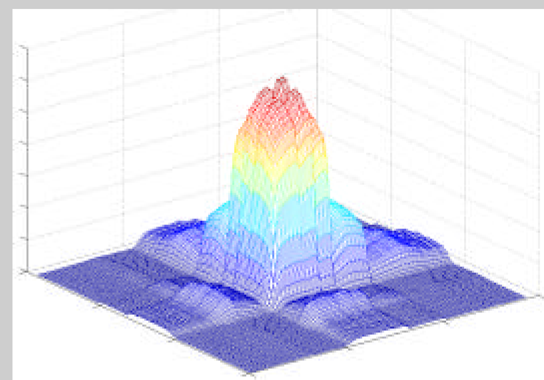



- HYDICE (HYperspectral Digital Imagery Collection Experiment)
  - Airborne sensor
- 210 spectral bands
  - 399-2501 nm
  - Channel widths ~ 3 – 11 nm
  - Spatial resolution, 1m x 1m
- Look for sub-pixel targets

Mean Target/Tree Spectra




Covariance for Trees



# Comparative Detector Performance
## Sub-pixel Targets

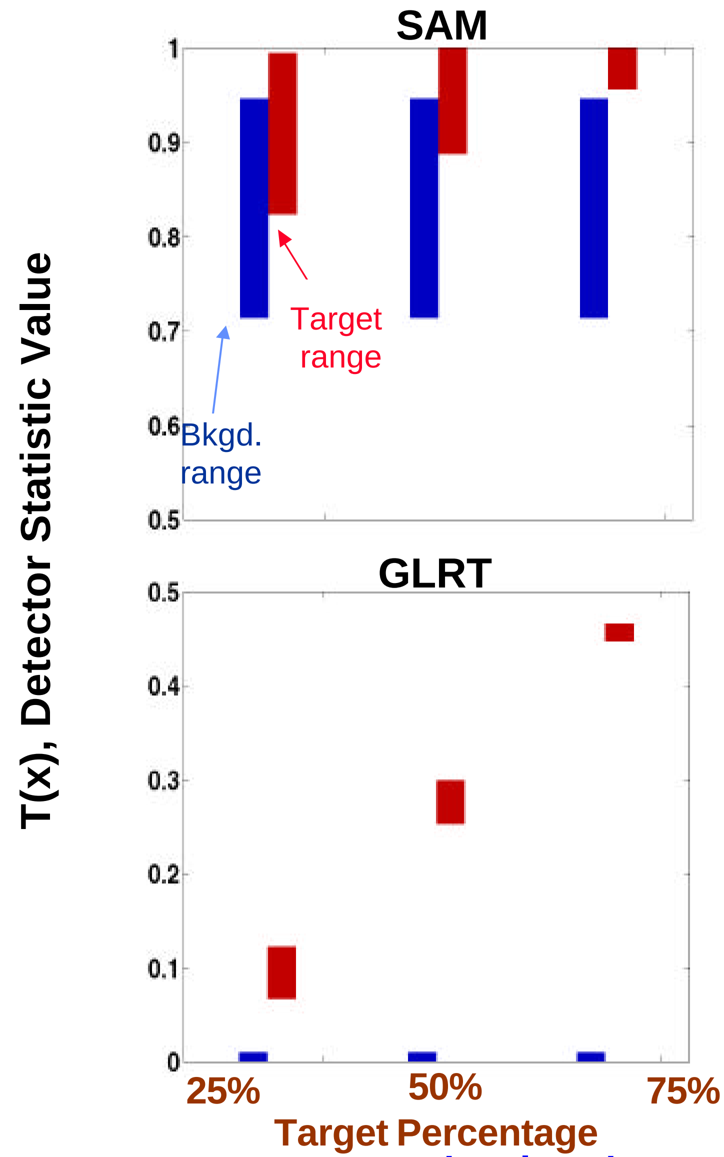
- **8232 tree pixels**
- **8232 synthetic mixed pixels**
  - 25% / 75%
  - 50% / 50%
  - 75% / 25%
- **Two detectors**
  - SAM ("unwhitened")

$$T_{SAM}(\mathbf{x}) = \frac{(\mathbf{s}^T\mathbf{x})}{\sqrt{(\mathbf{s}^T\mathbf{s})}\sqrt{(\mathbf{x}^T\mathbf{x})}}$$

  - GLRT

$$T_{GLRT}(\mathbf{x}) = \frac{(\mathbf{s}^T\overline{R_b}^{-1}\mathbf{x})^2}{(\mathbf{s}^T\overline{R_b}^{-1}\mathbf{s})(1+\mathbf{x}^T\overline{R_b}^{-1}\mathbf{x})}$$

  - Measure range of test statistics



# Conclusions

- Under LMM, hyperspectral sensing shares a common signal model with MTI radar
  – Endmembers « Steering vectors
  – Abundances « RCS
- Hyperspectral processing has leveraged optimal detection algorithms from radar
  – Exploit spectral differences between targets and background
- Successful sub-pixel target detection depends upon
  – Target/background subspace relationship
  – Fraction of target present